Problem 1.1

The Fahrenheit temperature scale is defined so that ice melts at 32°F and water boils at 212°F.

(a) Derive the formulas for converting from Fahrenheit to Celsius and back.

(b) What is absolute zero on the Fahrenheit scale?

Solution

Formulas converting between temperature scales are linear functions, so the conversion formula from Celsius to Fahrenheit has the form,

$$(°F) = m(°C) + b,$$

where m and b are constants to be determined. Ice melts at 0°C, and water boils at 100°C. As a result, two points on the line are $(0, 32)$ and $(100, 212)$. Use them to obtain a system of equations involving m and b.

$$32 = m(0) + b$$

$$212 = m(100) + b$$

Solving this system yields $b = 32$ and $m = 9/5$. Therefore, the formula for the Fahrenheit temperature given a Celsius temperature is

$$(°F) = \frac{9}{5}(°C) + 32. \tag{1}$$

Subtract 32 from both sides

$$(°F) - 32 = \frac{9}{5}(°C)$$

and multiply both sides by 5/9 to obtain the formula for the Celsius temperature given a Fahrenheit temperature.

$$(°C) = \frac{5}{9}[(°F) - 32] \tag{2}$$

Absolute zero in Celsius is -273.15°C. Plug this into equation (1) to get the corresponding Fahrenheit temperature.

$$(°F) = \frac{9}{5}(-273.15°C) + 32 = -459.67°F$$